

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Engineering Mathematics – I

Subject Code : 4TE01EMT3

Branch: B.Tech (All)

Semester : 1

Date : 21/03/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) n th derivative of $y = \frac{1}{x+a}$ is
- (A) $\frac{(-1)^n n!}{(x+a)^{n+1}}$ (B) $\frac{(-1)^{n-1} n!}{(x+a)^{n+1}}$ (C) $\frac{(-1)^n n!}{(x+a)^n}$ (D) none of these
- b) If $y = \cosh 2x$ and if n is even, then y_n equal to
- (A) $2^n \sinh 2x$ (B) $2^n \cosh 2x$ (C) $\cosh(2nx)$ (D) none of these
- c) If $y = \log(1+x)$, then x equal to
- (A) $1+y + \frac{y^2}{2!} + \frac{y^3}{3!}$ (B) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$ (C) $y + \frac{y^2}{2!} + \frac{y^3}{3!} + \frac{y^4}{4!} + \dots$
- (D) none of these
- d) If $y = \sin^{-1} x$, then x equal to
- (A) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$ (B) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$ (C) $1 - y + \frac{y^2}{2!} - \frac{y^3}{3!} + \dots$
- (D) $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$
- e) $\lim_{x \rightarrow \infty} x \left(a^{\frac{1}{x}} - 1 \right) = \underline{\hspace{2cm}}$
- (A) $\log_e a$ (B) 0 (C) 1 (D) none of these
- f) $\lim_{x \rightarrow \infty} \frac{x^n}{e^{ax}} = \underline{\hspace{2cm}}$
- (A) -1 (B) 0 (C) 1 (D) none of these
- g) If $x = r \cos \theta$, $y = r \sin \theta$, then
- (A) $\frac{\partial x}{\partial r} = \frac{\partial r}{\partial x}$ (B) $\frac{\partial x}{\partial \theta} = 0$ (C) $\frac{\partial x}{\partial r} = 0$ (D) $\frac{\partial x}{\partial r} = \frac{1}{\partial r / \partial x}$



- h) If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to
 (A) $\frac{\partial f / \partial x}{\partial f / \partial y}$ (B) $\frac{\partial f / \partial y}{\partial f / \partial x}$ (C) $-\frac{\partial f / \partial y}{\partial f / \partial x}$ (D) $-\frac{\partial f / \partial x}{\partial f / \partial y}$
- i) If $f_1 = \frac{vw}{u}$, $f_2 = \frac{wu}{v}$, $f_3 = \frac{uv}{w}$; then $\frac{\partial(f_1, f_2, f_3)}{\partial(u, v, w)}$ is equal to
 (A) 0 (B) 1 (C) 3 (D) none of these
- j) $\frac{\partial(u, v, w)}{\partial(x, y, z)} \cdot \frac{\partial(x, y, z)}{\partial(u, v, w)} = \text{_____}$
 (A) 2 (B) 1 (C) 0 (D) none of these
- k) If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i \sin\left(\frac{\pi}{2^r}\right)$ then $x_1 x_2 x_3 \dots$ to ∞ is
 (A) -3 (B) -2 (C) -1 (D) 0
- l) If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then
 (A) $a = 2, b = -1$ (B) $a = 1, b = 0$ (C) $a = 0, b = 1$ (D) $a = -1, b = 2$
- m) The system of equations $x + 2y + 3z = 1$, $x - y + 4z = 0$, $2x + y + 7z = 1$ has
 (A) exactly one solution (B) only two solutions (C) no solution
 (D) infinitely many solutions
- n) The product of the eigenvalues of $\begin{bmatrix} 1 & 4 \\ 2 & 10 \end{bmatrix}$ is
 (A) 2 (B) 4 (C) 6 (D) 0

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a) If $y = \frac{1}{x^2 + a^2}$ then find y_n . (5)
- b) Expand $e^{\sin x}$ as a series of ascending power of x upto x^4 . (5)
- c) If $u = \tan^{-1}\left(\frac{y}{x}\right)$ then verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. (4)

Q-3 Attempt all questions (14)

- a) If $y = \sin(m \sin^{-1} x)$ then prove that (5)
 $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$
- b) Prove that $\frac{x}{e^x - 1} = 1 - \frac{1}{2}x + \frac{1}{12}x^2 - \dots$ (5)
- c) Evaluate: $\lim_{x \rightarrow a} \log\left(2 - \frac{x}{a}\right) \cot(x - a)$ (4)

Q-4 Attempt all questions (14)

- a) Evaluate: $\lim_{x \rightarrow 0} \frac{a}{x^2} \left[\frac{\sin kx}{\sin lx} - \frac{k}{l} \right]$ (5)



b) If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$, evaluate $J = \begin{pmatrix} x, y \\ u, v \end{pmatrix}$ and $J' = \begin{pmatrix} u, v \\ x, y \end{pmatrix}$ and hence verify that $JJ' = 1$. (5)

c) Calculate approximate value of $\sqrt{9.12}$ by using Taylor's theorem. (4)

Q-5

Attempt all questions

(14)

a) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = -\frac{4}{(x+y)^2}$. (5)

b) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x + d^x}{4}\right)^{\frac{1}{x}}$ (5)

c) If $y = \sin^4 x$ then find y_n . (4)

Q-6

Attempt all questions

(14)

a) The power consumed in an electric resistor is given by $P = \frac{E^2}{R}$ (in watts). If (5)

$E = 200$ volts and $R = 8$ ohms, by how much does the power change if E is decreased by 5 volts and R is decreased by 0,20 ohms?

b) Prove that $\left(\frac{1 + \sin \theta + i \cos \theta}{1 + \sin \theta - i \cos \theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i \sin\left(\frac{n\pi}{2} - n\theta\right)$ (5)

c) Find the rank of matrix $A = \begin{bmatrix} 6 & 1 & 1 & 1 \\ 16 & 1 & -1 & 5 \\ 7 & 2 & 3 & 0 \end{bmatrix}$. (4)

Q-7

Attempt all questions

(14)

a) Using matrix method, show that the equations (5)

$$3x + 3y + 2z = 1, \quad x + 2y = 4, \quad 10y + 3z = -2, \quad 2x - 3y - z = 5$$

Are consistent and hence find the solution.

b) Find the roots common to the equations $x^4 + 1 = 0$ and $x^6 - i = 0$. (5)

c) Find $\tanh x$ if $5 \sinh x - \cosh x = 5$. (4)

Q-8

Attempt all questions

(14)

a) Find the inverse of $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ by Gauss-Jordan reduction method. (5)

b) Using De Moivre's theorem, expand $\sin^8 \theta$ in a series of cosines of multiples of θ . (5)

c) Check whether the following set of vectors is linearly dependent or linearly independent: (4)

$$(1, 2, -1, 0), (1, 3, 1, 2), (4, 2, 1, 0), (6, 1, 0, 1)$$

